

# Banking Leverage Procyclicality: A Theoretical Model Introducing Currency Diversification

Justine Pedrono

WP 2015 - Nr 39

# Banking leverage procyclicality: a theoretical model introducing currency diversification.

Justine Pedrono

Aix-Marseille University (Aix-Marseille School of Economics), CNRS & EHESS

justine.pedrono@univ-amu.fr

July 13, 2015

Work in progress<sup>1</sup>

## ABSTRACT:

The brutal adjustments to global banks' balance sheets in the wake of the recent economic crisis have rekindled interest in the procyclicality of banking leverage. During economic bursts, the collateral value of banks decreases and their risk-taking capacity is reduced. Banks raise less funds and their leverage - defined as total assets over equity - goes down: the leverage is procyclical. The paper investigates the procyclicality of bank leverage when banks can borrow and invest in two different currencies, as with European banks. To the extent that shocks are asymmetric, we find that currency diversification of assets reduces the procyclicality of the leverage and that a floating exchange rate increases the risk-taking capacity of banks.

JEL classification: F3, F4, G15

Keywords: procyclical leverage, bank, currency diversification, financial acceleration, globalization.

---

<sup>1</sup>DRAFT PAPER: Please do not cite or circulate without the author's permission

# 1 Introduction

The procyclicality of bank leverage is related to models of financial accelerators developed by Bernanke and Gertler [1989] and Kiyotaki and Moore [1997]. Financial distress, characterized by large declines in asset prices, decreases banks' net worth and increases their funding costs. This results in an endogenous process which is a major factor in depressed economic activity.

The development experienced by banks pre-crisis, followed by the crisis downturn, have recently drawn attention to bank leverage adjustments. According to Adrian and Shin [2013], the leverage of banks, defined as the ratio of total assets to equity, is procyclical: it rises in good times and falls in downturns. This procyclicality has two sources. First, banks' balance sheets are marked-to-market. Thus, an improvement in economic activity increases their net worth. Second, banks are active in the management of their balance sheets: their equity remaining constant, they reallocate the increase in their net worth to additional borrowing and investment. The leverage therefore increases. Figure 1 illustrates balance sheet adjustment following an improvement in economic activity.

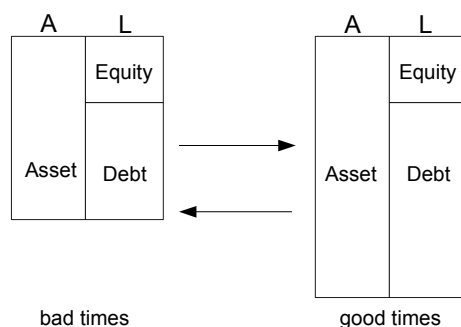


Figure 1: Procyclical leverage - Adrian and Shin (2013)

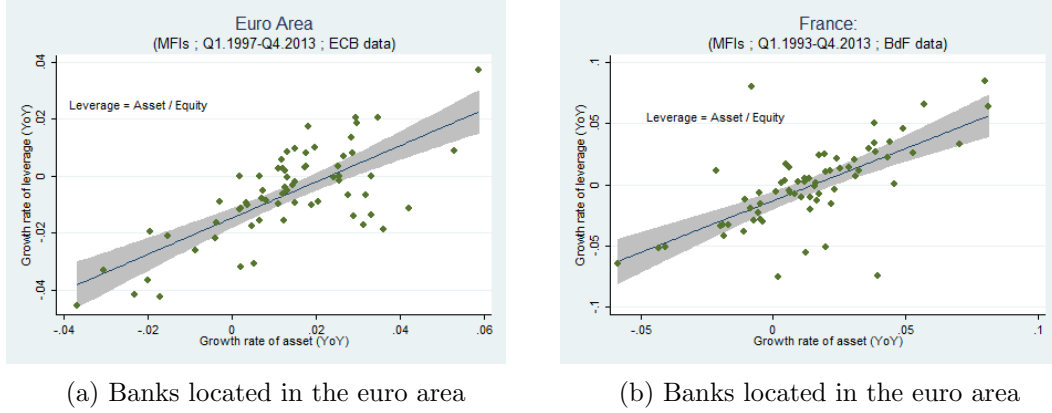


Figure 2: Procyclical leverage of banks, aggregate data.

As banks are active in the management of their balance sheet, their behavior is compatible with a Value at Risk (VaR) rule. Adrian and Shin [2013] build a micro-founded model which links leverage to the VaR rule. Banks adjust their balance sheets to maintain a given probability of failure.

Empirically, Adrian and Shin [2008], Gropp and Heider [2009], Kalemli-Ozcan et al. [2011], Baglioni et al. [2013] study the determinants of bank leverage and its alleged procyclicality. Only Gropp and Heider [2009] invalidate this relationship.<sup>2</sup> In the other papers, leverage is found to be especially procyclical for investment banks. Figure 2 supports this conclusion, plotting the growth rate of leverage along the growth rate of total assets. Both for banks located in the euro area (a) and banks located in France (b), the correlation is positive and significant.

Although the literature generally concludes that leverage is procyclical, it also highlights potential differences across geographic locations. According to Kalemli-Ozcan et al. [2011], European investment banks may show less procyclical leverage than US banks. This heterogeneity may reflect differences in the composition of banks' balance

---

<sup>2</sup>In Gropp and Heider [2009], banks' leverage converges to a bank-specific target.

sheets. As banks use their collateral to raise funds to finance investment, the composition of their collateral impacts leverage procyclicality. One major issue in this respect may be the currency denomination of the assets, which has not been incorporated in theoretical and empirical analyses on leverage procyclicality.

Adrian and Shin [2013] use a contracting model between a representative bank and its creditor which links the leverage to the domestic state of nature. Their model micro-founds the VaR Rule but excludes any currency diversification. More recently, Bruno and Shin [2015] introduce a cross-border network with a global bank, a regional bank and a local firms. Both the global and the regional bank perform their financial operations in foreign currency. In contrast, the local firm invests in local currency and raises debt from the regional bank in foreign currency. Thus, the risk connected with exchange rate fluctuations is only borne by the local firm and there is no currency diversification in the banks' balance sheets.

Since the early 2000s, banks have widely diversified both sides of their balance sheets. This diversification is related to the banks' international strategy, as highlighted by Baba et al. [2009], Borio and Disyatat [2011], Shin [2012], McGuire and Von Peter [2012]. Figure 3 gives a breakdown by currency of external banking positions based on BIS Data. On both sides of the balance sheet, the US dollar and the euro are the two major currencies used by reporting banks. Because they affect the value of banks' collateral, exchange rate variations should be incorporated in the analysis of leverage dynamics.

The purpose of this paper is to build a theoretical model which allows for currency diversification of both the assets and the liabilities of a bank. We extend the Adrian and Shin [2013] model by introducing a second currency of denomination on both sides of the balance sheet. The bank can borrow and invest in two different currencies: a

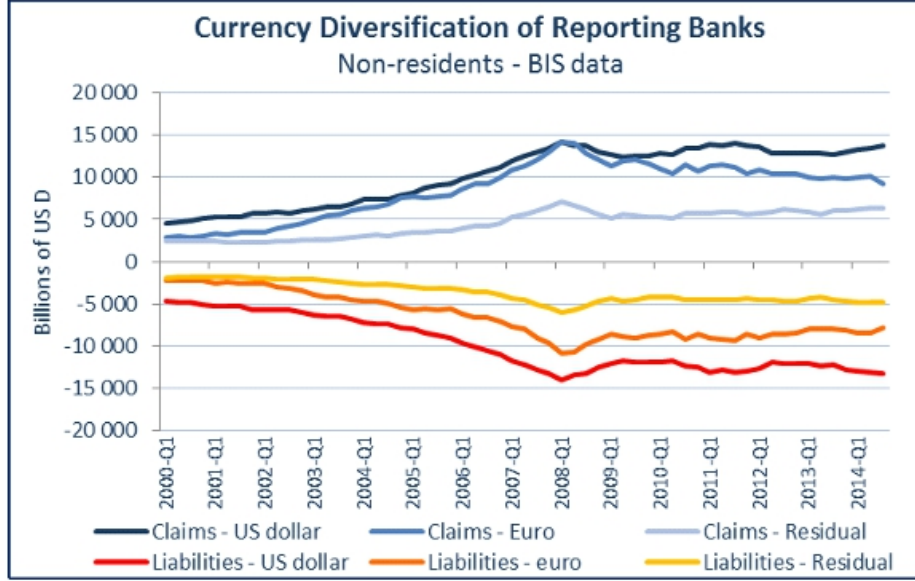


Figure 3: Currency breakdown of total reporting banks. International positions. BIS.

domestic currency which is the currency of the bank's equity, and a foreign currency. The bank's balance sheet is expressed in domestic currency, which implies a conversion of foreign assets and liabilities. Two exchange rate regimes are successively studied: a fixed regime and a floating regime, where the exchange rate depends on the relative state of nature in the two issuing countries.<sup>3</sup>

One important result from Adrian and Shin [2013] concerns the VaR rule. As banks follow a VaR rule, they adjust their balance sheet in order to maintain a constant probability of default. Introducing currency diversification does not affect the VaR rule or the mechanism behind the VaR rule. However, depending on the type of shocks and on the exchange rate regime, balance sheet adjustment will be affected by the degree of currency diversification. Three specific types of shocks are studied here: (i) a symmetric shock that does not affect the exchange rate; (ii) an anti-symmetric shock where the

<sup>3</sup>In contrast to Bruno and Shin [2015], the exchange rate here is directly linked to the relative state of nature.

domestic economy is positively impacted while the foreign economy is negatively impacted; (iii) and an asymmetric shock whereby both economies are affected in the same direction, but one more strongly than the other.

A positive shock in the home country induces a reaction of increased leverage. If the shock is symmetric, currency diversification of the balance sheet does not modify the extent of the leverage procyclical reaction, whatever the exchange rate regime. If the shock is anti-symmetric, the risk incurred on foreign assets induces a reaction of decreased of leverage, although less when the foreign currency depreciates. Finally, if the shock is asymmetric, procyclicality also diminishes with currency diversification.

The rest of the paper is organized as follows. Section 2 introduces the currency diversification in the Adrian and Shin [2013] framework. Section 3 develops the utility functions of agents. Two main constraints are derived from utility maximization. Section 4 defines the VaR rule and the reaction in terms of leverage to three economic shocks. Section 5 concludes and discusses some policy implications.

## 2 Currency diversification

The model is based on a representative bank's balance sheet. The bank invests in assets and raises funds from its creditors. There are two currency denominations for assets and debts, corresponding to two different countries. The economic states of nature corresponding to each economy are known publicly and determine the distribution of asset returns.

There are two periods  $T=0,1$ . Knowing the state of nature and the distribution of returns, the bank and the creditors agree on the amount to be reimbursed at  $T=1$  in order to satisfy the VaR rule. This amount therefore defines the level of debt the bank

is able to raise at  $T=0$ .

## 2.1 The accounting framework

The first agent in this model is a representative bank resembling a European investment bank. It is domestic in the sense that its equity and its balance sheet are in domestic currency. The domestic currency (e.g. the euro) is the primary currency. The bank is risk neutral and equity  $E$  is exogenous.<sup>4</sup> The second agent is the creditor of the bank, generally a Money Market Fund or another investment bank. The creditor lends money to the bank in both currencies. The creditor is also risk neutral. The exchange rate  $S$  is defined as the number of domestic units per unit of foreign currency.

At  $T=0$ , the bank raises funds backed by collateral in domestic and foreign currency ( $A$  and  $A^*$ , respectively). Total assets expressed in domestic currency are equal to  $A + SA^*$ . We denote by  $a$  the share of assets in domestic currency and  $(1 - a)$  the share of assets in foreign currency.  $a$  will vary depending on  $S$ .<sup>5</sup> Funds are in domestic and in foreign currency ( $D$  and  $D^*$ , respectively). Thus, total funding from creditors expressed in domestic currency is equal to  $D + SD^*$ .

At  $T=1$ , the bank receives a total expected return from its investments  $a(1 + \bar{r}) + (1 - a)(1 + \bar{r}^*)$ , where  $\bar{r}$  and  $\bar{r}^*$  are the expected returns from the domestic and the foreign asset, respectively. Returns depend on the state of nature specific to each currency area,  $\theta$  and  $\theta^*$ , respectively.  $\theta$  and  $\theta^*$  are known publicly from  $T=0$  and they do not change between the two periods.

At  $T=1$ , the bank reimburses its domestic and foreign creditors with respectively  $\bar{D}$  and  $S\bar{D}^*$ . As  $\theta$  and  $\theta^*$  are known for the two periods, there is no macroeconomic risk.

---

<sup>4</sup>An exogenous equity is in line with the theory of procyclical leverage put forward by Shin.

<sup>5</sup>See section 4.



However, the creditor of the bank faces a risk of default on repayment at  $T=1$ . The risk of default depends on the investment choice made by the bank. It is assumed that  $\bar{D} > D$  and  $S\bar{D}^* > SD^*$  to remunerate the creditor for the default risk. At  $T=0$ , the creditor receives a defaultable debt claim which becomes part of his/her utility function.

The bank's balance sheets at each period are given in table 1.

T=0, at market value:		T=1, at notional value:	
Assets	Liabilities	Assets	Liabilities
$A$	$E$	$(1 + \bar{r})A$	$\bar{E}$
$SA^*$	$D$	$(1 + \bar{r}^*)SA^*$	$\bar{D}$
	$SD^*$		$S\bar{D}^*$

Table 1: Bank's balance sheet at  $T=0$  and  $T=1$

## 2.2 Leverage

Four debt ratios are defined relative to each funding currency and each period. The debt ratios at  $T=0$  are:

$$d = \frac{D}{(A + SA^*)} \quad \text{and} \quad d^* = \frac{SD^*}{(A + SA^*)} \quad (1)$$

The corresponding notional values of debt ratios at  $T=1$  are:

$$\bar{d} = \frac{\bar{D}}{(A + SA^*)} \quad \text{and} \quad \bar{d}^* = \frac{S\bar{D}^*}{(A + SA^*)} \quad (2)$$

$\bar{E}$  is the equity at the notional value. It is equivalent to the equity at market value at  $T=0$  plus interest. In other words,  $E < \bar{E}$  and  $a(1 + \bar{r}) + (1 - a)(1 + \bar{r}^*) > (\bar{d} + \bar{d}^*)$ .

The leverage  $\lambda$  is defined as the ratio of total assets to equity, at market value:

$$\lambda = \frac{(A + SA^*)}{E} = \frac{(A + SA^*)}{(A + SA^*) - (D + SD^*)} = \frac{1}{1 - (d + d^*)} \quad (3)$$

### 2.3 Investment strategy

The bank makes an indivisible choice between two types of portfolio. Each portfolio is composed of an asset in domestic currency and an asset in foreign currency. The weight of each type of asset is given by  $a$  and  $(1 - a)$ . The portfolio's distribution comes from a mixture distribution of the two asset return distributions. As each asset return follows a General Extreme Value (GEV) distribution, the portfolio's return is also defined by a GEV distribution. The first portfolio is a good portfolio with a total expected return of  $[ar_H + (1 - a)r_{H^*}]$ , where  $r_H$  denotes the return from the good domestic asset and  $r_{H^*}$  the return from the good foreign asset. The second portfolio is not as good. Its total expected return  $[ar_L + (1 - a)r_{L^*}]$  is reduced through a parameter  $k$  (e.g  $k > 0$ ) and it involves more volatility through a parameter  $m$  (e.g  $m > 1$ ).<sup>6</sup> The cumulative distribution of the good asset in domestic currency, the cumulative distribution of the good asset in foreign currency, the cumulative distribution of the bad asset in domestic currency, and the cumulative distribution of the bad asset in foreign currency are the following, where  $\theta$ ,  $\sigma$  and  $\xi$  are respectively the location parameter, the scale parameter and the shape parameter, while  $z$  is the *iid* random variable:

---

<sup>6</sup>Introducing an investment choice enables a contract between the creditor and the bank to be modeled, as in Holmström and Tirole [1997].

$$\begin{aligned}
F_H(z) &= \exp \left\{ - \left( 1 + \xi \left( \frac{z - \theta}{\sigma} \right) \right)^{-\frac{1}{\xi}} \right\} \\
F_{H^*}(z) &= \exp \left\{ - \left( 1 + \xi \left( \frac{z - \theta^*}{\sigma} \right) \right)^{-\frac{1}{\xi}} \right\} \\
F_L(z) &= \exp \left\{ - \left( 1 + \xi \left( \frac{z - (\theta - k)}{\sigma m} \right) \right)^{-\frac{1}{\xi}} \right\} \\
F_{L^*}(z) &= \exp \left\{ - \left( 1 + \xi \left( \frac{z - (\theta^* - k)}{\sigma m} \right) \right)^{-\frac{1}{\xi}} \right\}
\end{aligned}$$

Using a mixture distribution, the Cumulative Distribution Function (CDF) of total return when the bank invests in the good portfolio is : <sup>7</sup>

$$F_{H,H^*}(z) = a \exp \left\{ - \left( 1 + \xi \left( \frac{z - \theta}{\sigma} \right) \right)^{-\frac{1}{\xi}} \right\} + (1 - a) \exp \left\{ - \left( 1 + \xi \left( \frac{z - \theta^*}{\sigma} \right) \right)^{-\frac{1}{\xi}} \right\} \quad (4)$$

If the bank invests in the bad portfolio, the CDF is defined by:

$$F_{L,L^*}(z) = a \exp \left\{ - \left( 1 + \xi \left( \frac{z - (\theta - k)}{\sigma m} \right) \right)^{-\frac{1}{\xi}} \right\} + (1 - a) \exp \left\{ - \left( 1 + \xi \left( \frac{z - (\theta^* - k)}{\sigma m} \right) \right)^{-\frac{1}{\xi}} \right\}$$

Thus, the total expected return of the portfolio depends on the type of shock the global economy faces (e.g  $\theta$  and  $\theta^*$ ).

The CDF allows us to define the probability of default  $\alpha$  when the bank invests in the good portfolio. This risk of default appears if the realized total return is below a given level equal to the total debt ratio at the notional value. Thus, the probability of

---

<sup>7</sup>This new framework using a mixture distribution is still compatible with a Second Order Stochastic Dominance, as in the reference model.

default  $\alpha$  is defined by the cumulative distribution function such that:

$$\begin{aligned}\alpha &= F_{H,H^*}(\bar{d} + \bar{d}^*) \\ &= a \exp \left\{ - \left( 1 + \xi \left( \frac{(\bar{d} + \bar{d}^*) - \theta}{\sigma} \right) \right)^{-\frac{1}{\xi}} \right\} + (1 - a) \exp \left\{ - \left( 1 + \xi \left( \frac{(\bar{d} + \bar{d}^*) - \theta^*}{\sigma} \right) \right)^{-\frac{1}{\xi}} \right\}\end{aligned}\tag{5}$$

Since the creditor is uninsured, he/she holds a defaultable debt claim with respect to the funds they lent to the bank at  $T=0$ . This claim will form part of the utility because it is part of the wealth at  $T=0$ . The value of this defaultable debt claim can be divided into two components: cash  $(\bar{D} + S\bar{D}^*)$  and a short position on a put option  $\pi$  (Merton [1974]). Since the risk differs between the two types of portfolio, the put option is specific to each investment choice.

If the bank invests in the good portfolio, we obtain the following put option price:<sup>8</sup>

$$\pi_{H,H^*}(\bar{D} + S\bar{D}^*, A + SA^*) = (A + SA^*) \cdot \pi_{H,H^*}(\bar{d} + \bar{d}^*, 1) \equiv (A + SA^*) \cdot \pi_{H,H^*}(\bar{d} + \bar{d}^*)$$

If the bank invests in the bad portfolio, the price of the put option is:

$$\pi_{L,L^*}(\bar{D} + S\bar{D}^*, A + SA^*) = (A + SA^*) \cdot \pi_{L,L^*}(\bar{d} + \bar{d}^*, 1) \equiv (A + SA^*) \cdot \pi_{L,L^*}(\bar{d} + \bar{d}^*)$$

### 3 Agents' participation constraints:

#### 3.1 Creditor's incentive constraint

The creditor of the bank is risk neutral. He maximizes his total net expected payoff.

---

<sup>8</sup>The price of the put option depends on the total amount reimbursed at the end of the period -  $\bar{D} + S\bar{D}^*$  - and on the total value of assets  $A + SA^*$ .

If the bank invests in the good portfolio, the net expected payoff is the following:

$$\begin{aligned} U_{H,H^*}^{c+c^*}(A + SA^*) &= (\bar{D} + S\bar{D}^*) - (A + SA^*)\pi_{H,H^*}(\bar{d} + \bar{d}^*) - (D + SD^*) \\ &= (A + SA^*) [(\bar{d} + \bar{d}^*) - \pi_{H,H^*}(\bar{d} + \bar{d}^*) - (d + d^*)] \end{aligned} \quad (6)$$

The requirement that utility is equal to or higher than 0 provides the first Participation Compatibility (PC) constraint:

$$0 \leq (\bar{d} + \bar{d}^*) - \pi_{H,H^*}(\bar{d} + \bar{d}^*) - (d + d^*) \quad (7)$$

$$(d + d^*) = (\bar{d} + \bar{d}^*) - \pi_{H,H^*}(\bar{d} + \bar{d}^*) \quad (\text{PC})$$

Similarly for an investment in the not so good portfolio:

$$\begin{aligned} U_{L,L^*}^{c+c^*}(A + SA^*) &= (A + SA^*) [(\bar{d} + \bar{d}^*) - \pi_{L,L^*}(\bar{d} + \bar{d}^*) - (d + d^*)] \\ (d + d^*) &= (\bar{d} + \bar{d}^*) - \pi_{L,L^*}(\bar{d} + \bar{d}^*) \end{aligned} \quad (8)$$

The PC constraints define the total debt ratio at market value relative to the total debt ratio at notional value. The latter should be large enough to form an incentive for the creditor to participate. The higher the reimbursement offered by the bank, the more the creditor is tempted to lend money at  $T=0$ . In this form, the incentive does not depend directly on the portfolio return specifications.

### 3.2 Bank's incentive constraint:

As the bank is risk neutral, it also maximizes its total net expected payoff. The introduction of a second investment currency changes the composition of the bank's net expected payoff  $U_{H,H^*}^B$ . In this framework, returns come from assets both in domestic and in foreign currency. Thus the net expected payoff when the bank invests in the good

portfolio is equal to:

$$\begin{aligned} U_{H,H^*}^B &= A.r_H + SA^*r_{H^*} + (D + S.D^*) - (\bar{D} + S.\bar{D}^*) + (A + SA^*).\pi_{H,H^*}(\bar{d} + \bar{d}^*) \\ &= (A + SA^*) [a.r_H + (1 - a)r_{H^*} + (d + d^*) - (\bar{d} + \bar{d}^*) + \pi_{H,H^*}(\bar{d} + \bar{d}^*)] \end{aligned} \quad (9)$$

When the bank invests in the bad portfolio the net expected payoff is equal to:

$$\begin{aligned} U_{L,L^*}^B &= A.r_L + SA^*r_{L^*} + (D + S.D^*) - (\bar{D} + S.\bar{D}^*) + (A + SA^*).\pi_{L,L^*}(\bar{d} + \bar{d}^*) \\ &= (A + SA^*) [a.r_L + (1 - a)r_{L^*} + (d + d^*) - (\bar{d} + \bar{d}^*) + \pi_{L,L^*}(\bar{d} + \bar{d}^*)] \end{aligned} \quad (10)$$

Assuming that  $U_{H,H^*}^B \geq U_{L,L^*}^B$  we get the Incentive Compatibility (IC) constraint:

$$a(r_H - r_L) + (1 - a)(r_{H^*} - r_{L^*}) \geq \pi_{L,L^*}(\bar{d} + \bar{d}^*) - \pi_{H,H^*}(\bar{d} + \bar{d}^*) \quad (11)$$

Where :  $(r_H - r_L) = (r_{H^*} - r_{L^*})$

$$r_H - r_L \geq \Delta\pi(\bar{d} + \bar{d}^*)$$

$$r_H - r_L = \Delta\pi(\bar{d} + \bar{d}^*) \quad (\text{IC})$$

The spreads in returns for all the currency denominations are similar. Thus, the left hand side (lhs) of the IC constraint can be simplified, as if the bank only held assets in the domestic currency.<sup>9</sup>

The IC constraint stipulates that for any economic condition there is a solution  $(\bar{d} + \bar{d}^*)$  that satisfies this identity. The unique solution illustrated in figure 4 comes from the Second Order Stochastic Dominance (SOSD) between the two mixture distributions and the differential in volatility.  $\pi(z)$  increases until  $F_{H,H^*}(z) = F_{H,H^*}(z)$  and decreases after the junction. As shareholders receive returns,  $(\bar{d} + \bar{d}^*) < (1 + \bar{r})$ , there is a unique solution  $(\bar{d} + \bar{d}^*)$  which satisfies the IC constraint.

---

<sup>9</sup>See the appendix.

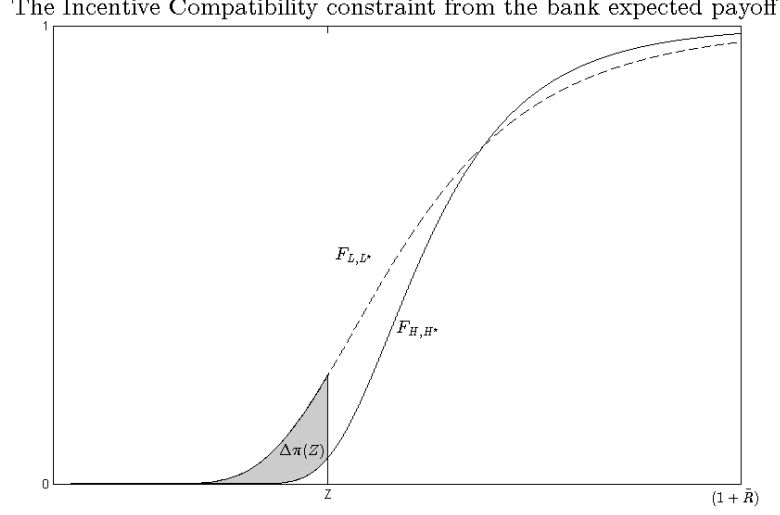


Figure 4: A unique solution to satisfy the IC constraint

As in Adrian and Shin [2013], the IC constraint also represents the moral hazard trade-off from Holmström and Tirole [1997]. The lhs represents the bank's private benefit from investing in the good portfolio while the right hand side (rhs) is equal to the private benefit from investing in the bad portfolio (e.g. low effort in the moral hazard model of Holmström and Tirole [1997]). With the added PC constraint from the creditor, the bank necessarily invests in the good portfolio where the put option induces lower prices.

However, in this form it is difficult to obtain a clear definition of  $(\bar{d} + \bar{d}^*)$ . Additional assumptions are needed to obtain a closed form solution.

### 3.3 Value at Risk

As in Adrian and Shin [2013], it is supposed here that  $\xi = -1$  and  $m \mapsto 1$ .<sup>10</sup> Thus, the CDF of the mixture functions are of the form:

$$\begin{aligned} F_{H,H^*}(z) &= a \exp \left\{ \frac{z - \theta}{\sigma} - 1 \right\} + (1 - a) \exp \left\{ \frac{z - \theta^*}{\sigma} - 1 \right\} \\ F_{L,L^*}(z) &= a \exp \left\{ \frac{z - (\theta - k)}{\sigma} - 1 \right\} + (1 - a) \exp \left\{ \frac{z - (\theta^* - k)}{\sigma} - 1 \right\} \end{aligned}$$

Hence  $F_{L,L^*} = e^{\frac{k}{\sigma}} F_{H,H^*}$

These assumptions allow the rhs of IC to be simplified as follows<sup>11</sup>

$$\begin{aligned} (r_H - r_L) &= \Delta\pi(\bar{d} + \bar{d}^*) \\ &= (e^{\frac{k}{\sigma}} - 1)\sigma F_{H,H^*}(\bar{d} + \bar{d}^*) \end{aligned} \tag{12}$$

Because  $F_{H,H^*}$  is the bank's probability of default when it invests in the good portfolio, we can extract the following VaR rule:

$$\alpha = F_{H,H^*}(\bar{d} + \bar{d}^*) = \frac{(r_H - r_L)}{(e^{k/\sigma} - 1)\sigma} \tag{13}$$

As the rhs of (13) does not depend on  $\theta$  or  $\theta^*$ , the VaR rule holds and the probability of default  $\alpha$  is maintained at the same level for any state of nature and any level of diversification. The bank adjusts its total debt ratio at T=1 in order to satisfy this identity.<sup>12</sup> Note that the VaR rule focuses on the tail of the distribution. If the tail is thickened by changes in the state of nature, the bank has to decrease its total debt ratio

---

<sup>10</sup> $\xi = -1$  implies a bounded distribution function on the right side. As the VaR rule focuses on the left side of the distribution, this assumption is not a problem.  $m \mapsto 1$  makes the volatility between the good and the bad asset comparable. It allows an approximation of a closed form solution.

<sup>11</sup>See the appendix.

<sup>12</sup>In a situation where the bank faces a positive shock and does not adjust its total debt ratio at T=1, the probability of default decreases. The bank receives greater returns, but does not reimburse more. To satisfy the VaR rule, the bank revises the debt ratio upwards.



in order to maintain  $\alpha$ .

Equation (13) is equivalent to:<sup>13</sup>

$$\alpha = aF_{H^*} + (1-a)F_{H^*} = \frac{(r_H - r_L)}{(e^{k/\sigma} - 1)\sigma} \quad (14)$$

Hence:

$$\alpha = \exp\left\{\frac{(\bar{d} + \bar{d}^*) - \theta}{\sigma} - 1\right\} \left[a + (1-a)\exp\left\{\frac{\theta - \theta^*}{\sigma}\right\}\right] = \frac{(r_H - r_L)}{(e^{k/\sigma} - 1)\sigma} \quad (15)$$

The VaR rule constrains the bank in its adjustment to the states of nature. The adjustment of  $(\bar{d} + \bar{d}^*)$  to the state of nature is:

$$(\bar{d} + \bar{d}^*) = \theta + \sigma + \sigma \ln\left(\frac{(r_H - r_L)}{(e^{k/\sigma} - 1)\sigma}\right) - \sigma \ln\left(a + (1-a)\exp\left\{\frac{\theta - \theta^*}{\sigma}\right\}\right) \quad (16)$$

The definition of the total debt ratio at the notional value is given by (16). If  $a = 1$ , there is no currency diversification and  $(\bar{d} + \bar{d}^*)$  depends on the domestic state of nature. In contrast, if  $a = 0$ , only the foreign state of nature affects  $(\bar{d} + \bar{d}^*)$ . The procyclicality of the leverage is derived from the degree of total debt ratio adjustment with respect to an economic shock.

**Proposition 1** *Currency diversification does not affect the VaR rule. The bank adjusts its balance sheet to the state of nature in both currency areas.*

---

<sup>13</sup>I use the following arrangement:  $F_{H^*} = F_H \cdot \exp\left\{\frac{\theta - \theta^*}{\sigma}\right\}$

## 4 Procyclical leverage with currency diversification

### 4.1 Economic shocks and exchange rate fluctuations:

As in Adrian and Shin [2013], return depends on the state of nature and on a function of the shape parameter  $H(\xi)$ .

The two economies in the model are assumed to be developed and similar, with perfect capital mobility. In a floating regime, the exchange rate  $S$  is determined based on the Uncovered Interest Rate Parity.<sup>14</sup>

**Hypothesis 1** *The definition of  $S$  follows the Uncovered Interest Rate Parity.*

$$S = 1 + \frac{r_{H^*} - r_H}{1 + r_H} \quad (17)$$

Where :

$$r_{H^*} = \theta^* + \sigma H(\xi)$$

$$r_H = \theta + \sigma H(\xi)$$

As  $\theta$  and  $\theta^*$  are known for both periods, the exchange rate does not change between  $T=0$  and  $T=1$ . Starting from an initial situation  $T < 0$  where  $\theta = \theta^*$ , a symmetric increase in  $\theta$  and  $\theta^*$  in the two economies does not change the interest rate spread. The exchange rate is maintained at its initial value  $S = 1$ . Now, if the amplitude of a positive shock is larger in the domestic economy, the domestic currency appreciates and  $S$  decreases. Because the domestic currency appreciates, the converted value of the foreign asset declines, which leads to a larger share of domestic assets relative to total assets:  $a$  goes up. Finally, an anti-symmetric shock such that  $\theta$  and  $\theta^*$  move in opposite directions

---

<sup>14</sup>The definition of  $S$  supposes that shocks are temporary.

but by the same amount adds to the depreciation of the foreign currency.  $S$  falls below unity and the rise in  $a$  is more pronounced.

Implicitly, we assume that the bank does not change the composition of its portfolio, notwithstanding the shock. Consequently, the changes in  $a$  and  $(1 - a)$  only reflect the exchange rate effect on converted value, or what we call the valuation effect of currency diversification. This makes it possible to clearly distinguish the impact of currency diversification on leverage.

**Hypothesis 2** *Changes in  $a$  and  $(1 - a)$  only reflect the exchange rate fluctuations.*

#### 4.2 A symmetric, positive shock:

If the two economies face a common positive shock, currency diversification does not affect the procyclicality of the leverage. Assets still offer a similar return and the exchange rate does not fluctuate. As illustrated in figure 5.a), the probability Density Functions (PDF) of the good portfolio total return shifts to the right. As total expected return goes up, the bank has to increase its total debt ratio  $(\bar{d} + \bar{d}^*)$  to maintain  $\alpha$  constant. The total debt ratio at the notional value goes from  $(\bar{d} + \bar{d}^*)_0$  to  $(\bar{d} + \bar{d}^*)_1$  in figure 5.b). The responsiveness of  $(\bar{d} + \bar{d}^*)$  to shocks is not different from that of a single currency framework.<sup>15</sup>

**Proposition 2** *Whatever the exchange-rate regime, currency diversification does not affect leverage procyclicality when shocks are symmetric.*

#### 4.3 An anti-symmetric shock:

The anti-symmetric shock is characterized by a positive shock domestically and an opposite shock in the foreign economy. The foreign asset is subject to increased risk of

---

<sup>15</sup>Proof in the appendix.

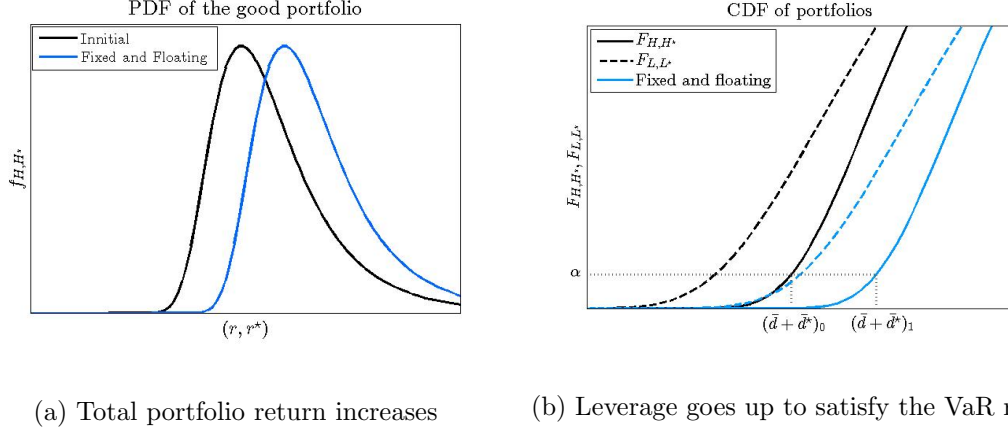


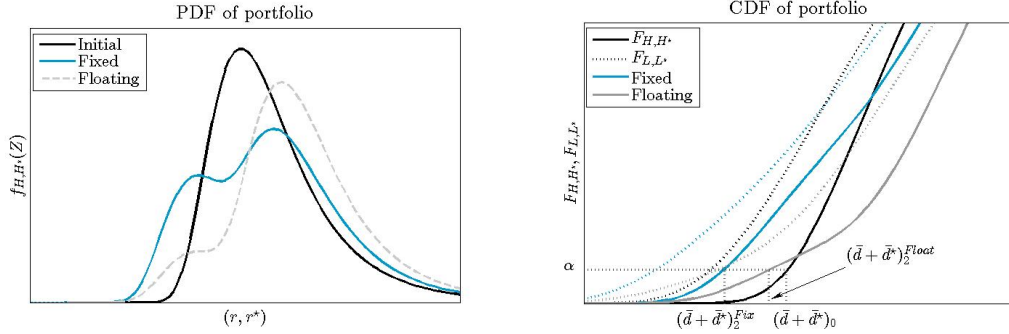
Figure 5: Global and positive shock: currency diversification does not affect leverage procyclicality

loss. As illustrated in blue in figure 6.a), the portfolio's PDF in a fixed exchange rate regime now includes two modes, one relative to each asset. As the portfolio held by the bank becomes riskier, the bank has to deleverage to satisfy the VaR rule and the constant probability of default  $\alpha$ . Figure 6.b) shows this new adjustment where  $(\bar{d} + \bar{d}^*)$  goes from  $(\bar{d} + \bar{d}^*)_0$  to  $(\bar{d} + \bar{d}^*)_2^{Fix}$ . Leverage becomes counter-cyclical.<sup>16</sup>

With an anti-asymmetric shock, the appreciation of the domestic currency is clearly visible. Therefore, the weight of domestic assets increases and the density relative to this mode goes up, as illustrated in grey in 6.a). As the risk of loss decreases, the portfolio becomes less risky than the fixed exchange rate regime. The bank still deleverages to satisfy the VaR rule and  $\alpha$ , but the adjustment is less brutal. Total debt ratio moves to  $(\bar{d} + \bar{d}^*)_2^{float}$ .

**Proposition 3** *In both exchange rate regimes, an anti-asymmetric shock leads to counter-cyclical leverage when there is currency diversification.*

<sup>16</sup>Proof in the appendix.



(a) Risks in portfolio increase

(b) Leverage still goes down to satisfy the VaR rule

Figure 6: Anti-symmetric shock: a counter-cyclical shock

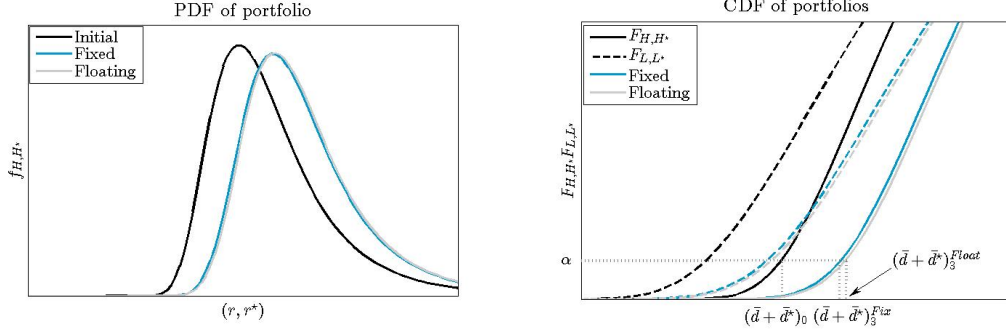
#### 4.4 An asymmetric and positive shock:

This last shock is positive but the amplitude of the shock is lower in the foreign economy. Thus, the return on assets differs and the distribution of the portfolio return flattens when the exchange rate is fixed. As illustrated in Figure 7, the blue PDF of the portfolio still shifts to the right but the density relative to the mode decreases. Consequently, the bank still increases  $(\bar{d} + \bar{d}^*)$  but to a lesser extent than in the symmetric case. It reaches  $(\bar{d} + \bar{d}^*)_3^{Fix}$  to keep  $\alpha$  constant in 7.b). Leverage procyclicality is reduced.<sup>17</sup>

In a floating regime, procyclicality increases compared to the fixed regime. The asymmetric shock leads to the appreciation of the domestic currency. As the converted value of the foreign asset decreases, the domestic asset weight in the portfolio increases. In Figure 7.a) the grey PDF moves slightly to the right compared to the fixed exchange rate regime. Thus, bank leverage has to be more procyclical to satisfy the VaR rule and the constant  $\alpha$  in figure 7.b). Total debt ratio rises from  $(\bar{d} + \bar{d}^*)_0$  to  $(\bar{d} + \bar{d}^*)_3^{float}$ .

Regarding the two economic shocks mentioned above, a floating exchange rate regime always promotes the asset which offers a better return. As the bank follows a VaR rule, the floating exchange rate regime increases its capacity to raise funds.

<sup>17</sup>Proof in the appendix.



(a) Total portfolio return increases less      (b) Leverage still goes up to satisfy the VaR rule

Figure 7: Asymmetric and positive shock: currency diversification reduces leverage procyclicality

**Proposition 4** *The introduction of a floating exchange rate increases the fund-raising capacity of the banks when shocks are anti-asymmetric or asymmetric.*

## Conclusion

Global banks follow global strategies regarding the composition of their assets and liabilities, with marked regional diversification of balance sheets. According to the empirical literature, this diversification may have an impact on leverage procyclicality. However, no account has previously been taken of currency diversification, which may affect the converted value of foreign assets in the balance sheet.

This paper offers the first theoretical model to introduce currency diversification in global banks' balance sheets. Based on Adrian and Shin [2013], the model micro-founds the VaR rule and confirms the active behavior of banks in response to economic shocks. Thus, depending on the type of shock and on the exchange rate regime, we find that currency diversification affects leverage adjustment. When shocks are asymmetric, leverage procyclicality can be reduced if foreign asset returns are less impacted by global financial cycles. When shocks are anti-asymmetric, the leverage becomes counter-cyclical in

both exchange rate regimes. When shocks are not correlated, the floating exchange rate regime promotes the asset which offers a better return. As the bank follows a VaR rule, the floating exchange rate regime increases its risk-taking capacity.

Two policy implications can be derived from these results. First, as currency diversification is not neutral, regulators should monitor the degree of currency diversification in addition to geographic diversification. Second, regulators could encourage diversification with assets less correlated to global financial cycles. This would reduce leverage procyclicality whatever the exchange-rate regime.

## 5 Appendix

### .1 Constant spreads:

As assets only differ in their location parameters, the spreads in interest rate are equal and constant relative to economic conditions.

$$\begin{aligned}
& a(r_H - r_L) + (1 - a)(r_{H^*} - r_{L^*}) \\
&= a.(\theta + \sigma H(\xi) - (\theta - k) - m\sigma H(\xi)) + (1 - a)(\theta^* + \sigma H(\xi) - (\theta^* - k) - m\sigma H(\xi)) \\
&= a.(k - \sigma(m - 1)H(\xi)) + (1 - a)(k - \sigma(m - 1)H(\xi)) \\
&= (k - \sigma(m - 1)H(\xi)) \\
&= Cst
\end{aligned} \tag{18}$$

### .2 IC development:

The simplifying assumptions give the following IC constraint:

$$\begin{aligned}
(r_H - r_L) &= \Delta\pi(\bar{d} + \bar{d}^*) \tag{IC} \\
&= \int_0^{\bar{d} + \bar{d}^*} F_{L,L^*} dz - \int_0^{\bar{d} + \bar{d}^*} F_{H,H^*} dz \\
&= e^{\frac{k}{\sigma}} \int_0^{\bar{d} + \bar{d}^*} F_{H,H^*} dz - \int_0^{\bar{d} + \bar{d}^*} F_{H,H^*} dz \\
&= (e^{\frac{k}{\sigma}} - 1) \int_0^{\bar{d} + \bar{d}^*} F_{H,H^*} dz \\
&= (e^{\frac{k}{\sigma}} - 1) \sigma F_{H,H^*}(\bar{d} + \bar{d}^*)
\end{aligned} \tag{19}$$

### .3 Proofs of leverage adjustments:

The general expression of total debt ratio at the notional value is the following:

$$(\bar{d} + \bar{d}^*) = \theta + \sigma + \sigma \ln \left( \frac{(r_H - r_L)}{(e^{k/\sigma} - 1)\sigma} \right) - \sigma \ln \left( a + (1 - a) \exp \left\{ \frac{\theta - \theta^*}{\sigma} \right\} \right)$$



Initially, the two economies are similar (e.g  $\theta_0 = \theta_0^*$ ) and total debt ratio is:

$$(\bar{d} + \bar{d}^*)_0 = \theta_0 + \sigma + \sigma \ln \left( \frac{(r_H - r_L)}{(e^{k/\sigma} - 1)\sigma} \right)$$

When economic shocks are symmetric and positive,  $\theta_1 = \theta_1^*$  and total debt ratio is:

$$(\bar{d} + \bar{d}^*)_1 = \theta_1 + \sigma + \sigma \ln \left( \frac{(r_H - r_L)}{(e^{k/\sigma} - 1)\sigma} \right)$$

Thus, leverage procyclicality is the following:

$$(\bar{d} + \bar{d}^*)_1 - (\bar{d} + \bar{d}^*)_0 = \theta_1 - \theta_0$$

Compared to Adrian and Shin [2013] leverage procyclicality is unchanged. When shocks are symmetric, whatever the exchange-rate regime, currency diversification does not affect leverage procyclicality.

Counter-cyclical leverage is observed when:

$$\begin{aligned} (\bar{d} + \bar{d}^*)_1 - (\bar{d} + \bar{d}^*)_0 &< 0 \\ \theta_1 - \theta_0 - \sigma \ln \left( a + (1-a) \exp \left\{ \frac{\theta_1 - \theta_1^*}{\sigma} \right\} \right) &< 0 \\ \ln \left( \frac{1}{(1-a)} \right) (\theta_1^* - \theta_0) &< \ln \left( \frac{a}{(1-a)} \right) (\theta_1^* - \theta_1) \end{aligned}$$

when the exchange-rate regime is fixed,  $a = (1-a)$  and the condition becomes:

$$\theta_1^* < \theta_0$$

This condition is satisfied when shocks are anti-symmetric. Anti-symmetric shocks lead to counter-cyclical leverage.

More generally, leverage procyclicality is reduced when:

$$\begin{aligned}
 (\bar{d} + \bar{d}^*)_1 - (\bar{d} + \bar{d}^*)_0 &< \theta_1 - \theta_0 \\
 a + (1 - a) \exp \left\{ \frac{\theta_1 - \theta_1^*}{\sigma} \right\} &> 1 \\
 \theta_1 &> \theta_1^*
 \end{aligned}$$

## References

- T. Adrian and H. S. Shin. Liquidity and financial cycles. *BIS WP 256*, 2008.
- T. Adrian and H. S. Shin. Procyclical leverage and value-at-risk. *NBER WP 18943*, 2013.
- N. Baba, R. McCauley, and S. Srichander Ramaswamy. Us dollar money market funds and non-us banks. *BIS Quarterly Review*, 2009.
- A. Baglioni, E. Beccalli, A. Boitani, and A. Monticini. Is the leverage of european banks procyclical? *Empirical Economics*, 43:1251–1266, 2013.
- B. Bernanke and M. Gertler. Agency costs, net worth, and business fluctuations. *American Economic Review*, 79:14–31, 1989.
- C. Borio and P. Disyatat. Global imbalances and the financial crisis: Link or no link? *BIS WP 346*, 2011.
- V. Bruno and H. S. Shin. Cross-border banking and global liquidity. *Review of economic studies*, 82:535–564, 2015.
- R. Gropp and F. Heider. The determinants of bank capital structure. *ECB WP*, 2009.
- B. Holmström and J. Tirole. Financial intermediation, loanable funds, and the real sector. *Quarterly Journal of Economics*, 112:663–691, 1997.
- S. Kalemli-Ozcan, B. Sorensen, and S. Yesiltas. Leverage across firms, banks and countries: some evidence from banks. *NBER WP 17354*, 2011.
- N. Kiyotaki and J. Moore. Credit cycles. *Journal of political economy*, 105:211–248, 1997.
- P. McGuire and G. Von Peter. The us dollar shortage in global banking and the international policy response. *International Finance*, 15:155–178, 2012.

- R. Merton. On the pricing of corporate debt: the risk structure of interest rates. *Journal of Finance*, 29:155–178, 1974.
- H. S. Shin. Global banking glut and loan risk premium. *IMF Economic Review*, 60: 155–192, 2012.